Definition of Position, Velocity and Acceleration

Let's start with one piston. It travels from end of the stroke to the other. It is only free to move along it's stroke path. The motion is restricted in all other directions by the fact that it is contained in a cylinder.

Next, I'll define some motion concepts of linear motion:
d = delta, which I will define as "the change in"
p = position - Where the piston is at a given time (t).
v = velocity - the rate of change of the position with time = dp/dt
a = acceleration - the rate of change of velocity with time = dv/dt

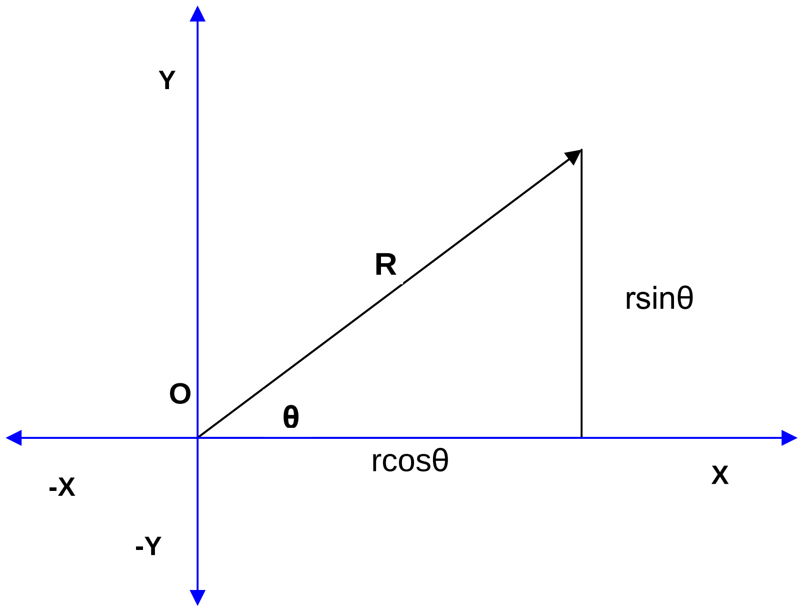
For ease of description, position, velocity and acceleration can be in either direction, positive or negative and maximum can mean maximum, or minimum.

As the piston reaches the end of the stroke (either end), it slows to a stop (v=0), then changes direction and accelerates in the other direction. At midstroke, the velocity reaches a maximum, then the piston accelerates in the other direction until it reaches the other end of the stroke, and velocity is again 0.

So when position (p) is maximum, velocity (v) is 0 and acceleration (a) is maximum in the opposite direction. When velocity is maximum, position and acceleration are zero.

I'm going to define a vector **R.** A vector is a quantity that has both a magnitude and a direction. The magnitude can be a distance, velocity, acceleration, force, etc. In this case, the magnitude is the distance from the centerline of the crankshaft main journal to the centerline of the connecting rod journal. This magnitude will be called r, which also corresponds to ½ of the stroke length. The vector rotates about the origin, **O**, at the intersection of the **X** and **Y** axes. We will assume a counter-clockwise (**X** to **Y**) rotation direction. The angle **θ** defines the direction of the **R** vector.

The X and Y axes are a fixed reference. They will form an unchanging reference. In this case they will also correspond to the centerlines of the piston cylinders. They are drawn here vertical and horizontal. They can be rotated to any orientation without changing any of the relationships. The 90 degree angle between cylinders is convenient for the analysis, but has other dynamic advantages, as well. We will define Piston 1 motion along the **Y** axis Piston 2 motion along the **X** axis.



Next we will define two terms, the sine (sin) and cosine (cos). These terms are a ratio between the length of the **R** vector at the angle θ, and the distance projected on to the **X** and **Y** axes. The distance projected along the **X** axis is rcosθ, and along the **Y** axis, it is rsinθ**.** These relationships are valid for any direction of the **R** vector. When the vector crosses the **Y** axis, the cos changes sign from positive to negative, or from a negative to a positive, depending on where the vector is pointing. The same is true for the sin when the vector crosses the **X** axis. This ability to break the vectors down into X and Y components is key to the analysis, as you will see.

So now, let's imagine that we rotate the coordinate system, so the **X** axis still points left and right across the screen, but the **Y** axis points directly out of the screen, toward you. Now, you can't see the sin values, only the cos values. These values correspond to the position of Piston 2 in its travel, except that the connecting rod is not shown yet, so it's just the relative position in the stroke, not the actual piston position. When the **R** vector crosses the **Y** axis, Piston 2 is at midstroke.

Now rotate the **Y** axis back where it was and rotate the axes, so the **Y** axis points up and down, but the **X** axis points out of the screen. Now you can only see the sin values, and these correspond to the position of Piston 1. When the **R** vector crosses the **X** axis, Piston 1 is at midstroke.

At this point I want to define another relationship, and that is the ratio of the distance across a circle, to the distance around it. The distance around a circle is always π (pi) times the distance across it. It so happens that π has the approximate value 3.14. So it's always π times farther around a circle than it is across it. π times the radius gets you half-way around a circle. So a dimensionless unit, radians, has been defined to define the arc length inscribed by a rotating radius vector, when the angle is defined in radians. The arc length is $r\theta$, when θ is defined in radians. This will all come in to play later.

Previously, I said that when a piston is at the end of the stroke (either end), the velocity is 0, the position is max (top or bottom) and the acceleration is max in the other direction. When the piston is at midstroke the position and acceleration are zero and the velocity is max. In beween these positions, the position, velocity and acceleration transition smoothly form 0 to max, or from max to zero.

Based on this, you can say that the velocity vector **V** lags the position vector **P** by 90 degrees ($\pi/2$ radians). The acceleration vector **A** lags the velocity vector by another 90 degrees (a total of π radians). And you will recall that velocity **V** is the rate of change of position, $d\mathbf{P}/dt$, and the acceleration, **A** is the rate of change of velocity, $d\mathbf{V}/dt$.

If you'll notice, the cos lags the sin by $\pi/2$. And if you go another $\pi/2$, you arrive at –sin. This is a very important concept when converting from position to velocity to acceleration and back.

We have describe position, velocity and acceleration with reciprocating motion, along a straight line. Next we need to define position, velocity and acceleration with circular motion. As defined above, the position of the rod journal is defined by the vector **R**. The actual position, for a given angle θ, is $r\theta$ where r is the length of **R**.

Next we will define the angular velocity, ω, which is the rate of change of the angle θ, or $d\theta/dt$. So the rate of change of of the angular position of the rod journal is $r\omega$, or $rd\theta/dt$. And finally, the angular acceleration, α, which is the rate of change of the angular velocity, $d\omega/dt$. So finally, the rate of change of angular velocity is $r\alpha$, or $rd\omega/dt$.

So, here we have described the basic relationship between reciprocating and rotating motion. Next we can get into the equations of motion.